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Generalized Minimum Variance Gain Scheduling Controller for Nonlinear Structural Systems under Seismic Ground Motion

It has been shown in the literature that the control of nonlinear structural systems using linear control techniques may not be effective. The permanent displacement of the structure due to material nonlinearity and damage may be reduced but not completely removed. In this paper we present a control method based on a linearized model of the structure in a predetermined desired states. Around each working state, a local Generalized Minimum Variance Control (GMV) is derived and applied. This method is commonly known as the Gain scheduling technique. We have used the decision algorithm for what local controller must be activated or disactivated. Simulation tests are performed using a single-degree-of-freedom nonlinear structure. The above mentioned approach shows to be effective in reducing the dynamic response and preventing the permanent displacement.

Keywords: generalized minimum variance control; nonlinear structures; gain scheduling

1. Introduction

Real world structural buildings often exhibit nonlinear behavior even if they are assumed to be well described by a linear model. The nonlinearity in structures is caused either by large displacements or material nonlinearity and damage. The problem is that most structural control techniques are designed assuming a linear behavior of the structure to be controlled [9]. These techniques may be efficient if the structural response remains within the linearly elastic range. But its performances will be severely degraded if the structure reaches the nonlinear range, especially in the case of severe earthquakes which causes large displacements and damage. In other hand, traditional techniques for analysis and synthesis of nonlinear controllers exist, but they are limited to specific classes of nonlinear systems.

Many researchers have addressed the problem of nonlinear structural control. Bani-Hani et al [3] used the potential of neural networks to control a nonlinear 3DOF structure. They demonstrated that the nonlinearly trained neurocontroller was able to reduce the structural damage and prevent the permanent displacement more than the linearly trained neurocontroller. In [17] the authors combined both base isolation and feedback control applied to a distributed parameter system. The nonlinear control consisted on a modified on-off with a two-tiered dead zone. An adaptive control algorithm based on sliding mode concepts for nonlinear uncertain dynamical systems is developed by Ghanem et al [14]. Yang et al [16] have applied the instantaneous on-line optimal control algorithm in nonlinear structural control.

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In this paper, a generalized minimum variance algorithm using the gain scheduling technique is presented. First, we linearize the nonlinear model of the structure around a number of desired states representing regions of evolution of the structural response. Around each state, an Auto-Regressive Moving Average- eXogen (ARMAX) model of the structure is determined and a local GMV control law is developed. The control consists on switching-on or switching-off a local GMV controller depending on the actual state of the system. This procedure (overbalancing between controllers when the state evolves from a region of linearization to another) is commonly known as the gain scheduling technique.

The approach is simulated using a SDOF nonlinear structure and the effectiveness is demonstrated in reducing the amplitude of vibrations and completely preventing the permanent displacement of the structure.

2. Nonlinear structural model

In this section, we are interested in formulating the dynamical equations of motion of a nonlinear single-degree-of-freedom structure under seismic excitation. In order to establish the dynamical model, the following assumptions are considered [5]:

1. the structure is supposed to be a lumped mass m in the girder
2. the two vertical axes are weightless and inextensible in the vertical direction with spring constant k/2 each.

Application of force equilibration principle to the nonlinear SDOF system shown in figure 1 gives

$$\ddot{x}(t)+2\xi\omega_0\dot{x}(t)+\alpha\omega_0^2x(t)+(1-\alpha)\omega_0^2z(t)=\frac{1}{m}u(t)-\ddot{x}_g(t) \qquad (1)$$

where m is the structural mass

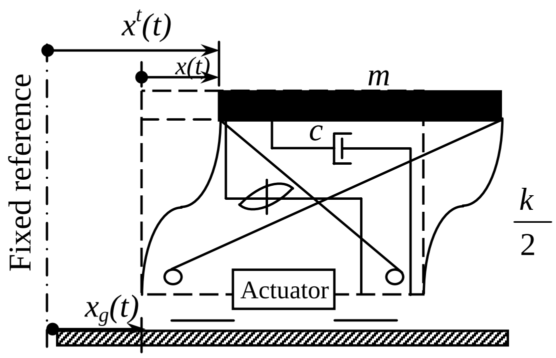


Figure 1. Single-degree-of-freedom structure

$\omega_0=\sqrt{k/m}$ is the linear natural frequency of the structure, with k the elastic stiffness

$\xi = c/2m\omega_0$ the damping ratio, with c the internal viscous damping of the structure

α is the rigidity ratio

$x(t)$ is the relative displacement

$\ddot{x}_g(t)$ is the ground acceleration

$u(t)$ is the external control force

z is the non linear hysteretic displacement defined by the following first order non-linear differential equation [5]

$$\dot{z}=h(z)\left(\frac{\dot{x}-\nu(\beta|\dot{x}||z|^{n-1}z+\gamma\dot{x}|z|^n)}{\eta}\right) \qquad (2)$$

where β, γ, n, η, ν and $h(z)$ are parameters that define the hysteretic behavior. More details on these parameters can be found in reference [5].

In this paper, we are interested on a simple nonlinear model called the Bouc-Wen original model, obtained from equation (2) by setting $h(z)$=1, $\nu=\eta$=1 and n=1. Thus from herein, the nonlinear displacement is defined by

$$\dot{z} = \dot{x} - \beta|\dot{x}|z - \gamma\dot{x}|z| \tag{3}$$

Because of the versatility and mathematical tractability of the Bouc-Wen model and its extensions, it has been widely used and applied to a variety of engineering problems, including SDOF and MDOF structural buildings. Although this model violates some plasticity postulates based on conservation laws [3], it has been shown to give accurate results.

By choosing the state space vector $X^T = [x_1 \;\; x_2 \;\; x_3] = [x \;\; \dot{x} \;\; z]$, equations (1) and (3) may be written in the following form

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -2\xi\omega_0 x_2 - \alpha\omega_0^2 x_1 - (1-\alpha)\omega_0^2 x_3 + \dfrac{1}{m}u - \ddot{x}_g \\ \dot{x}_3 = x_2 - \beta|x_2|x_3 - \gamma x_2|x_3| \end{cases} \tag{4}$$

3. ARMAX Model of the structure

To formulate an optimal control problem, it is necessary to specify the process dynamics and its environment. It is assumed that the influence of the environment on the process can be characterized by disturbances, which are stochastic process. As the system is nonlinear, we first linearize its model around a desired state. Then, using the principle of superposition we can represent all the disturbances as a single disturbance acting on the output. It is assumed that this disturbance is a stationary Gaussian process with rational spectral density.

The ARMAX (Auto Regressive Moving Average eXogen) model is used to represent the effect of both the control and disturbances on the system output. This model is well suited for stochastic optimal control problems [ ].

The state space equation (4) may be written in the following linearized form

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -2\xi\omega_0 x_2 - \alpha\omega_0^2 x_1 - (1-\alpha)\omega_0^2 x_3 + \dfrac{1}{m}u - \ddot{x}_g \\ \dot{x}_3 = a x_2 + b x_3 \end{cases} \tag{5}$$

where

$$\begin{cases} a = \left.\dfrac{\partial \dot{x}_3}{\partial x_2}\right|_{x_0} = \{1 - \beta x_3 sgn(x_2) - \gamma|x_3|\}_{x_0} \\ b = \left.\dfrac{\partial \dot{x}_3}{\partial x_3}\right|_{x_0} = \{-\beta|x_2| - \gamma x_2 sgn(x_3)\}_{x_0} \end{cases}$$

x_0 indicates the state around which the linearization is made.

Using equations (5), we ca deduce the continuous ARMAX model given by

$$X(s) = \frac{\overline{B}(s)}{\overline{A}(s)} U(s) + \frac{\overline{C}(s)}{\overline{A}(s)} \ddot{X}_g(s) \tag{6}$$

where $X(s), U(s)$ et $\ddot{X}_g(s)$ are the Laplace transforms of $x(t), u(t)$ et $\ddot{x}_g(t)$ respectively.

$$\begin{cases} \overline{A}(s) = s^3 + (2\xi\omega_0 - b)s^2 + \left(\alpha\omega_0^2 - 2b\xi\omega_0^2 + (1-\alpha)\omega_0^2 a\right)s - b\alpha\omega_0^2 \\ \overline{B}(s) = \dfrac{1}{m}(s-b) \\ \overline{C}(s) = (s-b) \end{cases} \tag{7}$$

Depending on the model of seismic excitation, different ARMAX models can be obtained, and the following cases arise

The seismic excitation model is unknown or is not taken into consideration. Thus, the earthquake acceleration is supposed to be a white noise excitation. The discrete ARMAX model of the structure is obtained directly by discretization of equation (7). Discretization techniques are readily available in computer programs (MATLAB 5.2)

$$x(t) = \frac{B(q^{-1})}{A(q^{-1})} u(t) + \frac{C(q^{-1})}{A(q^{-1})} \ddot{x}_g(t) \tag{8}$$

where

$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + a_3 q^{-3}$

$B(q^{-1}) = b_1 q^{-1} + b_2 q^{-2} + b_3 q^{-3}$

$C(q^{-1}) = c_1 q^{-1} + c_2 q^{-2} + c_3 q^{-3}$

q^{-1} shift operator defined as $q^{-1}x(t+1) = x(t)$

The polynomial parameters can be obtained by analytical discretization of equation (7) using the Z-transform.

4. Generalized minimum variance control

The Generalized Minimum Variance (GMV) algorithm was introduced by Clarke [4],[5] to control non-minimum phase systems. It is an extension of the Minimum Variance algorithm [1][2] which, by choosing a certain performance criterion, attempts to minimize the variance of the output.

The ARMAX model of the system is used

$$A(q^{-1})y(t) = q^{-d} B(q^{-1})u(t) + C(q^{-1})e(t) \tag{9}$$

where

$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_n q^{-n}$

$B(q^{-1}) = b_1 q^{-1} + \ldots + b_m q^{-m}$

$C(q^{-1}) = 1 + c_1 q^{-1} + \ldots + c_l q^{-l}$ noted *A*, *B* and *C*

$d \geq 0$ is the time delay of the system

$y(t)$ process output

$u(t)$ control

$e(t)$ white noise with zero mean and of variance σ^2.

The polynomial *C* is stable.

The performance index to be minimized is

$$J = E\left[\left(Py(t+d+1) - R_w w(t+d+1)\right)^2 + \left(Q' u(t)\right)^2\right] \tag{10}$$

where

E mathematical ensemble average

$w(t+d+1)$ reference signal

$P(q^{-1})$, $R_w(q^{-1})$ and $Q'(q^{-1})$ weighing polynomials with

$$P(q^{-1}) = \frac{P_N(q^{-1})}{P_D(q^{-1})} \text{ and } Q'(q^{-1}) = \frac{Q'_N(q^{-1})}{Q'_D(q^{-1})}$$

The degrees of P and R_w can be chosen arbitrarily.

Using equation (9) in equation (10), we derive the GMV control strategy given by [9]

$$u(t) = \frac{P_D C R_w w(t+d+1) - Ry(t)}{P_D(S+QC)} \qquad (11)$$

where $Q(q^{-1}) = \dfrac{q'_{N_0}\, p_{D_0}}{q'_{D_0}\, p_{N_0} b_1} Q'(q^{-1})$

q'_{N_0} , q'_{D_0} , p_{D_0} and p_{N_0} are the first coefficients of the polynomials Q'_N, Q'_D, P_D and P_N respectively.

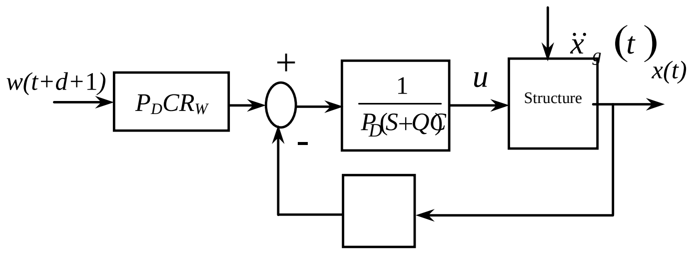


Figure 2. Generalized Minimum Variance control architecture

5. Generalized minimum variance control using gain scheduling

The gain scheduling strategy is a nonlinear control approach, which permits the extension of classical linear control algorithms to nonlinear systems. It is based on the linearization of the controlled process in a number of desired states. Then a linear controller is synthesized around each state based on the linearized model. The control consists of using operating one of these linear controllers depending on the evolution of the state of the controlled process. A schematic representation of this control strategy is shown in figure 3.

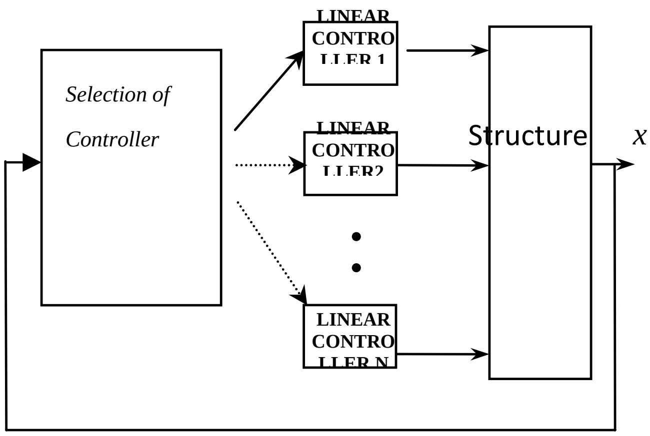


Figure 3. The gain scheduling Strategy

1) In our case this control strategy is implemented as follows

(1) linearize the nonlinear structural model in a number of chosen states. Then divide the state space into operating domains surrounding the states of linearization
(2) In each domain, and based on the linearized model, derive an ARMAX model of the structure, then a GMV controller is synthesized
(3) Operate the ith GMV controller when the state of the system is in the corresponding domain

6. Mathematical model of earthquake ground motion

The earthquake ground acceleration is modeled as a uniformly modulated non-stationary random process [7,9]

$$\ddot{x}_g(t) = \psi(t)\ddot{x}_s(t) \tag{12}$$

where $\psi(t)$ is a deterministic nonnegative envelope function and $\ddot{x}_s(t)$ is a stationary random process with zero mean and a Kanai-Tajimi power spectral density [7]

$$\phi_g(\omega) = \left[\frac{1+4\xi_g^2\left(\frac{\omega}{\omega_g}\right)^2}{\left(1-\left(\frac{\omega}{\omega_g}\right)^2\right)^2+4\xi_g^2\left(\frac{\omega}{\omega_g}\right)^2}\right]S_0^2 \tag{13}$$

where ξ_g, ω_g are filter parameters and S_0 is the constant spectral density of the white noise. However, it can be shown that the velocity and displacement spectra, which are derived from the acceleration spectra that are described by equation (16), have strong singularities at zero frequency. These singularities can be removed by using high-pass filter, as suggested by Clough-Penzien [6]. Using such a second high pass filter, the Kanai-Tajimi spectrum is modified as follows to obtain the Clough-Penzien spectrum [9]

$$\phi_c(\omega) = \left[\frac{1+4\xi_g^2\left(\frac{\omega}{\omega_g}\right)^2}{\left(1-\left(\frac{\omega}{\omega_g}\right)^2\right)^2+4\xi_g^2\left(\frac{\omega}{\omega_g}\right)^2}\right]\left[\frac{\left(\frac{\omega}{\omega_c}\right)^4}{\left(1-\left(\frac{\omega}{\omega_c}\right)^2\right)^2+4\xi_c^2\left(\frac{\omega}{\omega_c}\right)^2}\right]S_0^2 \tag{14}$$

A particular envelope function $\psi(t)$ given in the following, will be used

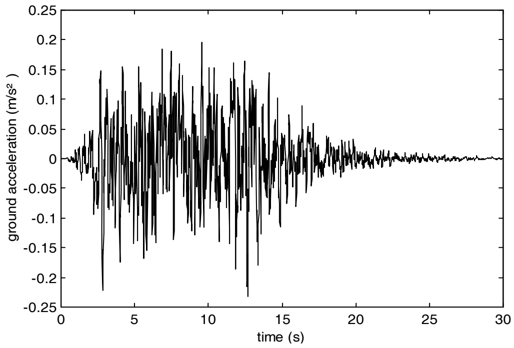
$$\psi(t) = \begin{cases} 0 & \text{for} \quad t<0 \\ \left(\frac{t}{t_1}\right)^2 & \text{for} \quad 0 \le t \le t_1 \\ 1 & \text{for} \quad t_1 \le t \le t_2 \\ \exp\left[-a(t-t_2)\right] & \text{for} \quad t \ge t_2 \end{cases} \tag{16}$$

where t_1, t_2 and a are parameters that should be selected appropriately to reflect the shape and duration of the earthquake ground acceleration.

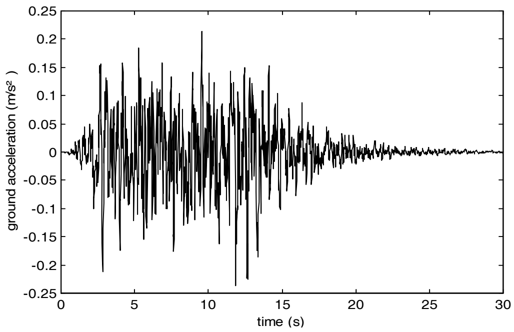
7. Simulation results

The Kanai-Tajimi and Clough-Penzien ground accelerations have been simulated and are presented in figure 4. Numerical values of parameters are t_1=3s, t_2=13s, a=0.26, ξ_g=0.65, ω_g=19rad/s, ξ_c=0.6, ω_c=2rad/s, S_0=0.8 10^{-2}m/s.

Sample hysteresis plot of the SDOF structure under white noise excitation with power spectral density S_0=10 is shown in figure 5. The following structural parameters are considered : m=2000 kg, ω_0=2 rad/s, ξ=0.05, α=0.1, β=1.5, γ=-0.5.



(a) Kanai-Tajimi model



(b) Clough-Penzien model

Figure 4. Simulated ground accelerations

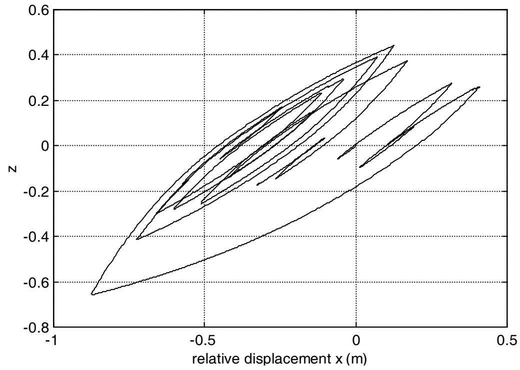


Figure 5. Sample hysteresis plot of nonlinear SDOF structure under white noise excitation

Simulation tests have been performed to evaluate the performances of the control approach presented in this paper, using the nonlinear SDOF structure described in section 2. We have linearized the structural model in 4 points corresponding to 4 operating domains.

To implement the GMV algorithm we have used a sampling period T_e =0.02s. parameters of ARMAX models. Ponderation polynomials are $P_N(q^{-1})=1-0.01q^{-1}$, $P_D(q^{-1})=1$, $Q(q^1)=5.10^{-6}$.

Responses of the structure to 300% of Kanai-Tajimi seismic excitation with S_0=1 are shown in figures 6,

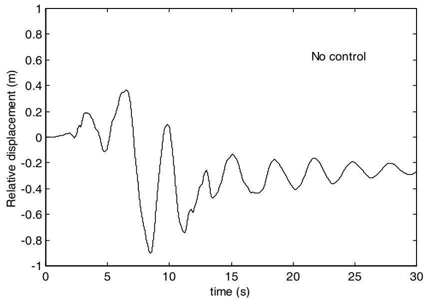


Figure 6.a The relative displacement

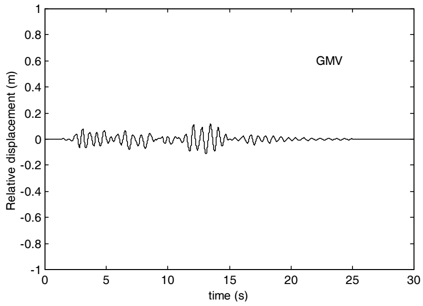


Figure 6.b The relative displacement

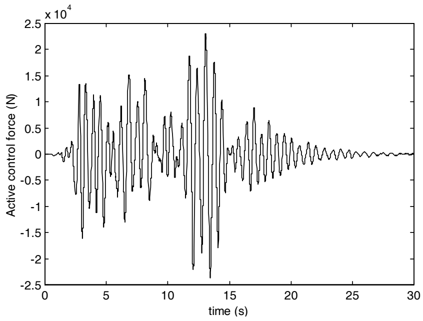


Figure 6.c The active control force

Figure 6. The gain scheduling approach

8. Conclusion

We have investigated in this paper a control approach, the gain scattering control, which permits the extension of linear control algorithms to nonlinear systems. Simulation results have shown the efficiency of this control technique in reducing the relative displacement and preventing the permanent displacement due to the nonlinear behavior of the structural system.

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